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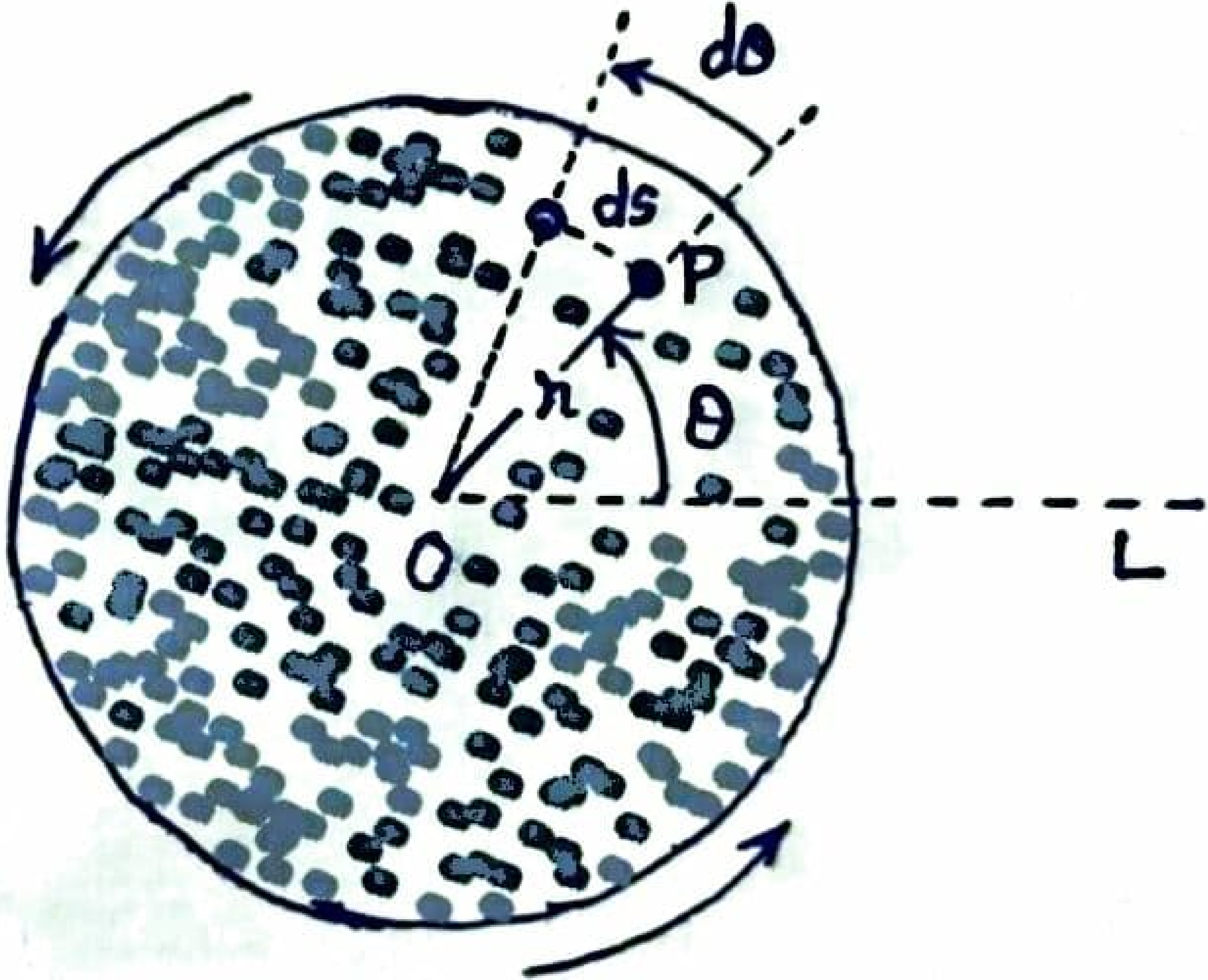
5. ROTATION ABOUT A FIXED AXIS.

1) KINEMATICS: ⇒

Consider an ith particle 'P', of a wheel rotating about a fixed-axis through-its-centre.
We can specify the position of P, by its distance r from the rotation axis, and the angle θ-in-the-wheel's plane measured anticlockwise from a fixed line OL, as shown in the following figure.



dθ
ds
P
r
θ
O
L

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In a small time dt, the particle P moves a distance ds along the arc of a circle of radius r. The angle dθ swept out by the line OP, in radian, is the distance ds divided by the radius r.

$$d\theta = ds/r$$

- Angular Position, Angular Velocity, & Angular Acceleration:

Although the distance ds varies from particle to particle, the angle dθ swept out in a given time, and therefore, the rate of change of angle, dθ/dt, is the same for all the particles of the wheel. It is called the angular velocity ω of the wheel.

$$\omega = d\theta/dt$$

Like linear velocity, angular velocity is also a vector. But we'll wait until the next chapter for the full vector description of ω. For now it (and so, dθ) is positive for anticlockwise rotation and negative for clockwise rotation. This is the same kind of description we used for linear velocity when we studied rectilinear motion.

The units of angular velocity are rad/s or simply, reciprocal seconds since radians are dimensionless. Although the angular velocity can be expressed in rps, rpm, or °/s, it is important to note that the expression like ω = dθ/dt and, the other results we obtain for rotational motion are valid only when 'angles' are expressed in radians. Note that, 1rev = 2π rad = 360°

The rate of change of angular velocity with respect to time is called the angular acceleration α of the wheel.

$$\alpha = d\omega/dt = \omega d\omega/d\theta = d^2\theta/dt^2$$

Angular acceleration has the same direction as angular velocity if the angular 'speed' is increasing, and the opposite direction if it is decreasing.

Because angular velocity and angular

acceleration are defined analogously to linear velocity and linear acceleration, all the relations among linear position, velocity, and acceleration automatically apply among angular position, angular velocity, and angular acceleration.
For example, if angular acceleration is constant, we have:

$$\omega' = \omega + \alpha t$$
$$\Delta\theta = \omega t + \alpha t^2/2$$
$$\omega'^2 = \omega^2 + 2\alpha\Delta\theta$$

Note that in case of constant angular velocity, $\alpha = 0$ and $\omega = 2\pi/T = 2\pi f$.

- Relation between Translational & Rotational Quantities:

From equation $d\theta = ds/r$, the speed v of the ith particle P of the wheel is related to the angular speed of the wheel as,

$$ds/dt = r.d\theta/dt$$
$$v = r\omega$$

The relation between the tangential acceleration of the ith particle P and the angular acceleration of the wheel is obtained by taking the derivative of the speed v in equation $v = r\omega$.

$$a'' = r\alpha$$

Each particle of the wheel also has a radial acceleration (centripetal acceleration) which points inward along the radial line, and has the magnitude

$$a' = v^2/r = r\omega^2$$

2) TORQUE ABOUT AN AXIS: ⇒

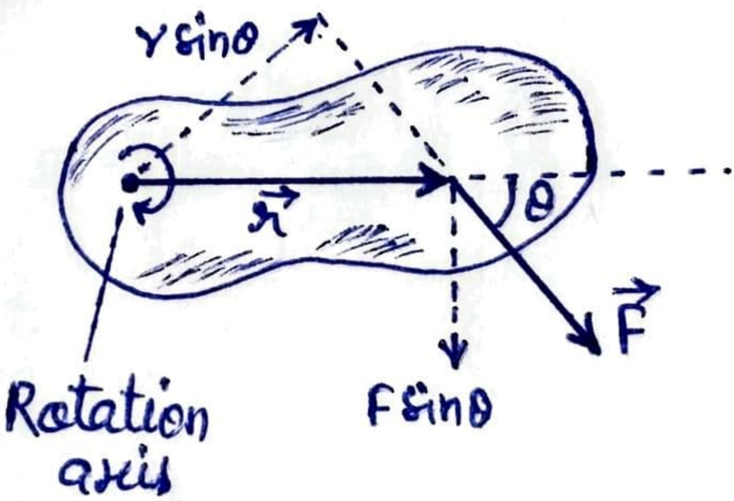
By sitting far from a seesaw's rotation axis, a small child can balance her father on the seesaw. That way, her smaller weight at a greater distance from the rotation axis, is as effective as her father's greater weight closer to the axis.

In general, this effectiveness (called torque) of an "axis-perpendicular" force in producing the rotation of an object about the axis, depends not only on the magnitude of the force and the position vector of the force-application point relative to the axis, but also on the angle θ between them. (See below figure).
It is greatest when the force F is perpendicular to r, and decreases to zero as they become colinear. Based on these observations, we define torque as the product of the distance r and the component of the force perpendicular to r.

$$\tau = F\sin\theta . r = FL$$

where $L = r\sin\theta$, called 'lever arm', is the effective distance from the axis at which F acts. We'll extend our notation of torque to provide its full vector description in the next chapter. For now we will specify its direction as positive for ACW, and negative for CW. Note that, a force which is coplanar with the rotation axis, doesn't have the ability to rotation the object njeev Mishra and 2 others >

about the axis, hence doesn't produce torque about the axis.



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3) THE ROTATIONAL ANALOGS OF: MASS, NEWTON'S LAW, WORK, POWER, KINETIC ENERGY, & MOMENTUM: ⇒

Ultimately Newton's second law governs all motions, but its application to a rotating object would be terribly cumbersome. We instead formulate an analogous law that deals with rotational quantities.

To develop such a law, we need rotational analogs of force, mass, and acceleration. Torque and angular acceleration are the rotational analogs of force and acceleration. We still need the rotational analog of mass, that is, we want a quantity that describes resistance to changes in rotational motion.

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• Newton's law:

Consider an “object” consisting of a “massless” rod of length r with a particle of mass m attached on the end. The object is free to rotate about a stationary axis perpendicular to the rod and passing through its free end. If we apply a force F to the particle always at right angles to the rod, the particle undergoes a tangential acceleration given by F = ma". We can express a" in terms of the angular acceleration α of the object, a" = rα. There's also a tension force in the rod, but it doesn't contribute to the torque, therefore the net torque about the rotation axis can be written as

$$\tau = Fr\sin 90° = ma''r = mr\alpha r$$

That is, $\tau = (mr^2)\alpha$

Here we have Newton's law F = ma, written in terms of rotational quantities. The torque, (analogous to force) is the product of the angular acceleration (analogous to acceleration) and the quantity mr^2, which must therefore be the rotational analog of mass. We call this quantity the rotational inertia or moment of inertia and give it the symbol I.

Given the rotational inertia I, our rotational analog of Newton's law becomes

$$\tau = I\alpha \qquad ...(A)$$

• Work:

For the rotational analogs of work, power, & kinetic energy, consider again the above object.

In a small time dt, as the particle moves a distance ds along the circle, the work done by F is, $dW = Fds$. Since $ds = rd\theta$, where $d\theta$ is the angle swept out by the rod, the work dW can be written as $dW = Frd\theta = \tau d\theta$

$$W = \int \tau d\theta \qquad ...(B)$$

• Power:

The rate of doing work is the power developed by the torque, $P = dW/dt = \tau d\theta/dt$. Since $d\theta/dt = \omega$, so

$$P = \tau\omega \qquad ...(C)$$

• Kinetic energy:

In terms of the angular speed ω, the kinetic energy of the particle becomes $k = m(r\omega)^2/2$, or

$$k = I\omega^2/2 \qquad ...(D)$$

Equation (A), (B), (C), & (D), are analogous to the similar results for rectilinear motion. Although we derived them for a single, localized mass, they apply to extended-mass objects if we interpret τ as the net torque on the object and I as the net rotational inertia of the object.

- Angular momentum:

We can write equation (A) in an alternative way $\tau = d(I\omega)/dt$. This looks like the linear equation $F = d(mv)/dt$ for rectilinear motion, where mv is the linear momentum. The quantity $I\omega$, which must therefore be the rotational analog of linear momentum, is called the angular momentum L of the object about the rotation axis. The rotational analog of Newton's law can then be

$$\tau = dL/dt$$

This equation is more general than equation $\tau = I\alpha$. For example, it applies to a system in which the rotational inertia is not constant.

We can also write, $\Delta L = \int \tau dt$. The quantity on the right side of this equation is called the angular impulse. This angular impulse momentum theorem is particularly effective in the impulsive motion, like collision. Specifically, if net torque τ is zero, L = constant, that is, $I_1\omega_1 = I_2\omega_2$.

Note that, angular momentum defined by the equation $L = I\omega$, is only the angular momentum "component" along the rotation axis. However, if the object is rotating about a symmetric axis passing through its CM, it is the total angular momentum too.

4) CALCULATING THE ROTATIONAL INERTIA: ⇒

An object's rotational inertia about an axis is the sum of the rotational inertias of the individual mass elements making up the object.

$$I = \sum mr^2$$

Here m is the mass of the ith element, and r is its distance from the axis.

With continuous mass,

$$I = \int r^2 dm$$

It is easy to see that an object's rotational inertia depends on the mass of the object as well as on the distribution of the mass relative to the axis.

We can often simplify the calculation of the rotational inertia by using two theorems:

(1) If we know the rotational inertia I about an axis through the CM of an object, the parallel-axis theorem allows us to calculate the rotational inertia I' through any parallel axis. The parallel-axis theorem states that

$$I' = I + Md^2$$

Where d is the distance from the CM axis to the parallel axis and M is the total mass of the object.

(2) An another theorem called perpendicular axis theorem, relates the

rotational inertias I_1 & I_2, about two perpendicular axes in the plane of a 'plane-object' to the rotational inertia I_3 about a third axis perpendicular to the plane of the object & passing through the interaction-point of the previous two axes. It states that

$$I_3 = I_1 + I_2$$

The rotational inertia is often given in handbooks in terms of the 'radius of gyration' k, defined by

$$I = Mk^2$$

Where M is the total mass of the object. The radius of gyration is a measure of the distribution of mass of an object relative to a given rotation axis. A large radius of gyration means that, on the average, the mass is relatively far from the axis.

- Rotational inertia of common objects.